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# NOVEL MULTISTAGE MULTIRATE SYSTEM FOR ECG SIGNAL

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## *ABSTRACT*

Multistage multirate signal processing is a qualifying technology that brings DSP techniques to the applications changing the rate of a signal. The Multirate filtering technique is widely used for meeting different sampling rates in the system. This paper envisages the implementation of multistage proposed filter structures of polyphase decomposition techniques with decimator and interpolator. This proposed filter structure is implemented using an analysis filter and synthesis filter with polyphase branches which reduces the computational complexity and adopts parallelism. The results analysis show in three different cases of proposed filter structure with multistage and compare with wavelet 9/7 filter to find the best removing noise elimination filter structure for ECG signal.

***KEYWORDS:*** *Multistage,Multirate, Analysis, Synthesis, ECG*

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## INTRODUCTION

Wavelet transforms have been used in the field of signal and image processing. Recently, research on wavelet construction called lifting scheme, has been established by Wim Sweldens and Ingrid Daubechies [1]. This is also referred to as wavelet 9/7 filter. This scheme was developed on FIR-based discrete transform. In this, the input signal is fed into a low pass filter and high pass filter separately. The outputs of the two filters are then sub sampled. The original signal can be reconstructed by synthesis filters *h* and *g* which take the up-sampled Lowpass and high pass inputs. This scheme is also used in the Laurent polynomial representation of the filter and Euclidean algorithm. These schemes shows some limitations on sampling methods.

Frequency converters sampling methods and narrow band filtering are known, allowing significant computational efficiency [2, 3]. However, current design procedures for these multistage and multilayer filters address the specification of each phase individually, rather than simultaneously optimizing all of the filter phases [4]. The authors [5] formulate an algorithm that optimizes multi-stage adaptive coefficients and also provides sufficient conditions for multi-track filter identification. A multilayer digital filter (MDF) is a digital filter that changes the input sampling frequency from the input signal to another signal. There are many applications in communication, image processing, digital audio and multimedia. In [6], the modern DSP system uses MDF with three factors. First of all, MDF is used in two digital systems with a different sampling rate. Second, MDF is the best approach to solving the complex filtering problem. Third, multilayer filtering is used in the construction of the multilayer filter bank.

Manually, constructing a multi-stage poly-phase filter (i.e., decimation (reduced sampling rate), filtered and polyphase interpolation (increased sampling frequency) is a time-consuming and high-risk process. which we focus on in this paper is the third factor.We developed the multistage multirate system support with multirate polyphase filters. This elimination process is easy for specific and computing a multistage multirate design. The main concept of proposed system, the input signal is decimated/interpolated and decomposed with $M^{th}$ polyphase filter branch with sampling rate conversion is called analysis filter in the first stage. In the second stage, the input signal is decomposed with Decimator and then the polyphase multistage filter by converting the sampling rate to the rational factor L/M that represents the test filter of the system proposed. This process is continued for another stage for getting more and more smooth signal.

## MATERIALS AND METHODS

### *Database*

In this paper, we need to import ECG data for analysis, and so used the MIT-BIH Atrial Fibrillation database [7]. This database includes different types of noise such as power line noise, baseline noise, EMG noise, abrupt shift noise and electrosurgical noise.

### Proposed MultistageMultirate Polyphase Filter

Figure 1 shows the general scheme of proposed multistage multirate system design. This system has been used the fractional sampling rate by cascading a factor-of-L interpolator with a factor-of-M decimator, where L and M are positive integer. We have used as H(z) decimation filter and G(z) as interpolation filter. These two filters operate with same or different sampling rates, they can be replaced with a single filter designed to avoid aliasing. The $1^{st}$ stage and $2^{nd}$ stage form an analysis filter and synthesis filter with polyphase branches. Configurations based on polyphase decomposition are convenient in applications where *L* and *M* are small numbers. Otherwise, filters of a very high order are requested. For proposed multistage Multirate filter system, design considers three cases which are described below.

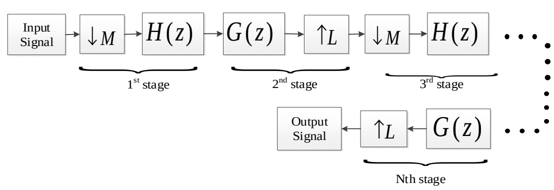



**Figure 1: Proposed diagram for Multistage Implementation of Sampling Rate Alteration System**

### Case 1

Consider the proposed design of low-pass FIR filter with poly-phase decomposition and decimation for ECG signal to meet the following specifications:

**Table 1**

| Parameters | Parameters Values |
|---|---|
| Decimate factor (M) | 5 |
| Pass-band edge | $0.08\pi$ |
| Stop-band edge | $\frac{2\pi}{M} - \omega_p$ |
| Pass-band ripples | 40 dB |
| Stop-bands ripples | 60 dB |

Designing the filter transfer function H(z) by Nth order Lowpass FIR digital filter from the above specifications and returns the filter coefficients in length as shown in Table I and its frequency response in Figure 2 where we take the sampling frequency. The transfer function is formed as

$$H(Z) = h[0] + h[1]z^{-1} + h[2]\, z^{-2} + h[3]z^{-3} + h[4]z^{-4} + h[5]\, z^{-5} + h[6]\, z^{-6} + h[7]z^{-7} + h[8]\, z^{-8} + h[9]z^{-9} \quad (1)$$

**Table 2: The Impulse Response h(n) or Filter Coefficients of Frequency Sampling Filter (N=9)**

| n | h[n] | N | h[n] |
|---|---|---|---|
| 0 | -0.0022 | 9 | -0.0022 |
| 1 | 0.0669 | 8 | 0.0669 |
| 2 | 0.1097 | 7 | 0.1097 |
| 3 | 0.1610 | 6 | 0.1610 |
| 4 | 0.1928 | 5 | 0.1928 |

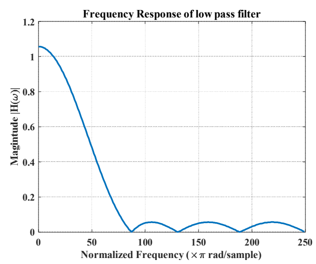



**Figure 2: Frequency Response of Design Low Pass for FIR Filter**

Then performed decomposition of the filter transfer function into 5 poly-phase components using the equation (2).

$$H(z) = E_0(z^5) + z^{-1}E_1(z^5) + z^{-2}E_2(z^5) + z^{-3}E_3(z^5) + z^{-4}\, E_4(z^5) \quad (2)$$

Where,

$$E_0(z) = \{h[0] + h[5]\, z^{-5}$$

$$E_1(z) = \{h[1] + h[6]\, z^{-5}\}$$

$$E_2(z) = \{h[2] + h[7]z^{-5}\}$$

$$E_3(z) = \{h[3] + h[8]\, z^{-5}\}$$

$$E_4(z) = \{h[4] + h[9]z^{-5}\}$$

So,

$$H(z) = \sum_{k=0}^{M-1} z^{-k} E_k(z^M) \quad \textbf{(3)}$$

Where,

$$E_k(z) = \sum_{n=0}^{[N/M]} h[Mn + k]z^{-n}. 0 < k < M - 1 \quad \textbf{(4)}$$

We set up decimate-by-M=5 for the input signal x[n]. In this step, the input {x[n]} is decomposed into the set of 5 subsequences: $\{x_0[\mathrm{m}]\}$, $\{x_1[\mathrm{m}]\}$, $\{x_2[m]\}$, $\{x_3[m]\}$, $\{x_4[m]\}$ Here, for the cause {x [n]} we have x [−1] = x[−2] = x[−3] = x[−4] = 0. The 5 subsequences are filtered in the poly-phase branches and added together to give the decimated signal $\{y_{dec}[m]\}$. The realization analysis filter structure of proposed system is shown in Figure 3.

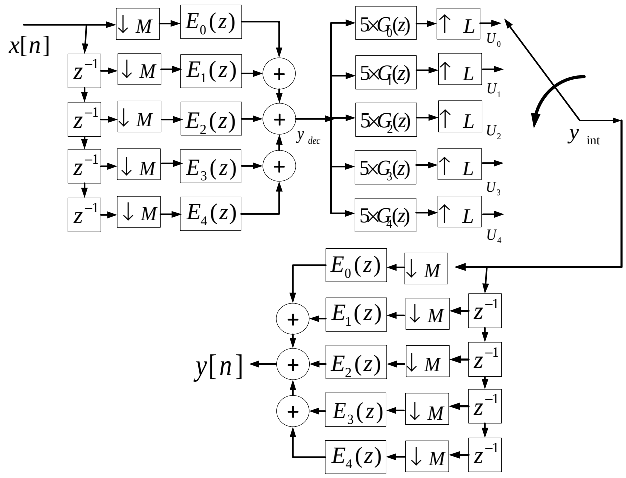



**Figure 3: Proposed Multistage Multirate System for Case 1**

We chose interpolation-by- L=5 into the multistage multirate system structure using polyphase decomposion as shown in Figure 3. In this step, signal $\{y_{dec}[m]\}$is used as an input to the interpreter.The signal {x[n]} = $\{y_{dec}[m]\}$ is filtered in the parallel poly-phase branches and the set of 5 signals ,$\{g_0[n]\}$, $\{g_1[n]\}$,$\{g_2[n]\}$, $\{g_4[n]\}$, $\{g_5[n]\}$, is obtained as the equations (8)-(12). The matrix G is composed of 5 row vectors: $\{g_0[\mathrm{n}]\}$, $\{g_1[\mathrm{n}]\}$, $\{g_2[\mathrm{n}]\}$, $\{g_3[\mathrm{n}]\}$, $\{g_4[\mathrm{n}]\}$. The samples of the interpolated signal $\{y_{int}[\mathrm{m}]\}$are stored column-wise in matrix U. The interpolated signal is considered as a synthesis filter is obtained simply by picking up the samples from matrix U.

$$g_0(n) = \{h[0]y_{dec}(n) + h[5]y_{dec}(n-1)\} \tag{5}$$

$$g_1(n) = \{h[1]y_{dec}(n) + h[6]y_{dec}(n-1)\} \tag{6}$$

$$g_2(n) = \{h[2]y_{dec}(n) + h[7]y_{dec}(n-1)\} \tag{7}$$

$$g_3(n) = \{h[3]y_{dec}(n) + h[8]y_{dec}(n-1)\} \tag{8}$$

$$g_4(n) = \{h[4]y_{dec}(n) + h[9]y_{dec}(n-1)\} \tag{9}$$

Taking inverse Z-transform of the equations (5) to (9), is obtained as

$$G_0(z) = \{h[0]Y[z] + h[5]z^{-1}Y[z]\} \tag{10}$$

$$G_1(z) = \{h[1]Y[z] + h[6]z^{-1}Y[z]\} \tag{11}$$

$$G_2(z) = \{h[2]Y[z] + h[7]z^{-1}Y[z]\} \tag{12}$$

$$G_3(z) = \{h[3]Y[z] + h[8]z^{-1}Y[z]\} \tag{13}$$

$$G_4(z) = \{h[4]Y[z] + h[9]z^{-1}Y[z]\} \tag{14}$$

## Case 2

In case 2, we considered a $N^{th}$ order low-pass digital FIR filter with poly-phase decomposition and decimation/interpolation for biomedical signals to meet the following specifications:

**Table 3**

| Parameters | Parameters Values |
|---|---|
| Decimate factor (M) | 5 |
| Filter order, N | 81 |
| Cut-off frequency | $1/M$ |

**Table 4: The Impulse Response h(n) or Filter Coefficients of Frequency Sampling Filter (N=81)**

| n | h[n] | n | h[n] |
|---|---|---|---|
| 0 | 0.00019 | 81 | 0.00019 |
| 1 | -0.00020 | 80 | -0.00020 |
| 2 | -0.00057 | 79 | -0.00057 |
| 3 | -0.00079 | 78 | -0.00079 |
| 4 | -0.00072 | 77 | -0.00072 |
| 5 | -0.00032 | 76 | -0.00032 |
| 6 | 0.00037 | 75 | 0.00037 |
| 7 | 0.00113 | 74 | 0.00113 |
| 8 | 0.00163 | 73 | 0.00163 |
| 9 | 0.00154 | 72 | 0.00154 |
| 10 | 0.00068 | 71 | 0.00068 |
| 11 | -0.00079 | 70 | -0.00079 |
| 12 | -0.00240 | 69 | -0.00240 |
| 13 | -0.00342 | 68 | -0.00342 |
| 14 | -0.00317 | 67 | -0.00317 |
| 15 | -0.00138 | 66 | -0.00138 |
| 16 | 0.00157 | 65 | 0.00157 |
| 17 | 0.00467 | 64 | 0.00467 |
| 18 | 0.00652 | 63 | 0.00652 |
| 19 | 0.00594 | 62 | 0.00594 |
| 20 | 0.00255 | 61 | 0.00255 |
| 21 | -0.00286 | 60 | -0.00286 |
| 22 | -0.00840 | 59 | -0.00840 |
| 23 | -0.01161 | 58 | -0.01161 |
| 24 | -0.01050 | 57 | -0.01050 |
| 25 | -0.00448 | 56 | -0.00448 |
| 26 | 0.00502 | 55 | 0.00502 |
| 27 | 0.01471 | 54 | 0.01471 |
| 28 | 0.02041 | 53 | 0.02041 |
| 29 | 0.01859 | 52 | 0.01859 |
| 30 | 0.00803 | 51 | 0.00803 |
| 31 | -0.00913 | 50 | -0.00913 |
| 32 | -0.02742 | 49 | -0.02742 |
| 33 | -0.03929 | 48 | -0.03929 |

**Table 4 Contd.,**

| n | h[n] | n | h[n] |
|---|---|---|---|
| 34 | -0.03741 | 47 | -0.03741 |
| 35 | -0.01717 | 46 | -0.01717 |
| 36 | 0.02128 | 45 | 0.02128 |
| 37 | 0.07245 | 44 | 0.07245 |
| 38 | 0.12641 | 43 | 0.12641 |
| 39 | 0.17140 | 42 | 0.17140 |
| 40 | 0.19695 | 41 | 0.19695 |

From the above sections, we found the transfer function H(z) by $N^{th}$ order Lowpass FIR digital filter and filter coefficients h[n] in length N+1 as shown in Table 4. Its frequency response in Figure 4 where fs=500Hz. The transfer function is formed as

$$H[z] = h[0] + h[1]z^{-1} + h[2]z^{-2} + \cdots + h[81]z^{-81} \quad \textbf{(15)}$$

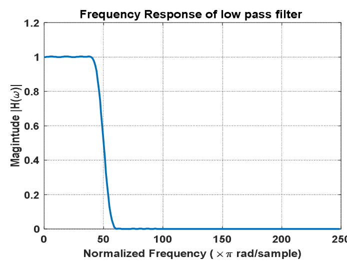



**Figure 4: Frequency Response of Design Low Pass for FIR Filter**

Then performed decomposition of the filter transfer function into 2 poly-phase components using the equation (15).

$$\mathrm{H}(z) = \mathrm{E}_0(z^2) + z^{-1}\mathrm{E}_1(z^2) \quad \textbf{(16)}$$

Where,

$$E_0 = h[0] + h[2]z^{-2} + h[4]z^{-4} + \cdots + \mathrm{h}[80]z^{-80}$$

$$E_1 = h[1]z^{-1} + h[3]z^{-3} + h[5]z^{-5} + \cdots + h[81]z^{-80}$$

$$E_{00} = h[0] + h[6]z^{-6} + h[12]z^{-12} + \cdots + h[78]z^{-78}$$

$$E_{01} = h[2]z^{-2} + h[8]z^{-8} + h[14]z^{-14} + \cdots + \mathrm{h}[80]z^{-80}$$

$$E_{02} = h[4]z^{-4} + h[10]z^{-10} + h[16]z^{-16} + \cdots + \mathrm{h}[76]z^{-76}$$

$$E_{10} = h[1]z^{-1} + h[7]z^{-7} + h[13]z^{-13} + \ldots + h[79]z^{-79}$$

$$E_{11} = h[3]z^{-3} + h[9]z^{-9} + h[15]z^{-15} + \ldots + h[81]z^{-81}$$

$$E_{12} = h[5]z^{-5} + h[11]z^{-11} + h[17]z^{-17} + \cdots + h[77]z^{-77}$$

Decimated-by-M=2. In this step, the input $\{x[n]\}$is broken down into the set of 2 subsequences: $\{x_0[m]\}$, $\{x_1[m]\}$. Here, for the causal {x[n]} we have x[-1] = x[-2] = 0. The 2 subsequences are interpolated by 3 in the poly-phase branches and added together to give the decimated signal $\{y_{dec}[m]\}$The actualization analysis filter structure of the proposed system is shown in Figure 5.

For multistage purpose, further we have up-sampled & down-sampled the decimated signal $\{y_{dec}[m]\}$several times, such as, M=40 & L=40, M=20 & L=20, M=20 & L=40, M=40 & L=20.

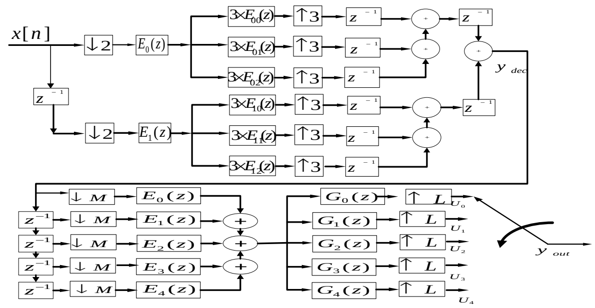



**Figure 5: Proposed Multistage Multirate System for Case 2**

## Case 3

Consider the design specification of case 2, the decimated signal $\{y_{dec}[m]\}$of case 2 is used as an input to the interpolator and interpolated by L=2, then the 2 subsequences are decimated by 3 in the poly-phase branches and added together to give the interpolated signal $\{y_{int}$ [m]}. The fulfillment analysis filter structure of proposed system is shown in Figure 6.

The signal $\{x[n]\} = \{y_{dec}[m]\}$is filtered in the parallel poly-phase branches and the set of 2 signals $\{\{g_0$ [n]}, $\{g_1$ [n]} and the subsequences of this two signals $\{\{g_{00}$ [n]}, $\{\{g_{01}$ [n]}, $\{\{g_{02}$ [n]} $\{\{g_{10}$ [n]}, $\{\{g_{11}$ [n]} $\{\{g_{12}$ [n]} is obtained as the equations (28)-(32) and added together to give the interpolated signal $\{y_{int}$ [m]}.

$$\{g_0[n]\} = \{g[0], g[M], g[2M], \ldots\}$$

$$\{g_1[n]\} = \{g[-1], g[M-1], g[2M-1], \ldots\}$$

$$\{g_2[n]\} = \{g[-2], g[M-2], g[2M-2], \ldots\}$$

$$\{g_{M-1}[n]\} = \{g[-M+1], g[1], g[M+1], g[2M+1], \ldots\} \quad (17)$$

Where the equation can be represented based on the design specifications as,

$$g_0[n] = h[0]y_{dec}[n] + h[2]y_{dec}[n-1] + h[4]y_{dec}[n-2] + \cdots + h[80]y_{dec}[n-40] \quad \textbf{(18)}$$

$$g_1[n] = h[1]y_{dec}[n] + h[3]y_{dec}[n-1] + h[5]y_{dec}[n-2] + \cdots + h[81]y_{dec}[n-40] \quad \textbf{(19)}$$

$$g_{00}[n] = h[0]y_{dec}[n] + h[6]y_{dec}[n-3] + h[12]y_{dec}[n-6] + \cdots + h[78]y_{dec}[n-39] \quad \textbf{(20)}$$

$$g_{01}[n] = h[2]y_{dec}[n-1] + h[8]y_{dec}[n-4] + h[14]y_{dec}[n-7] + \cdots + h[80]y_{dec}[n-40] \quad \textbf{(21)}$$

$$g_{02}[n] = h[4]y_{dec}[n-2] + h[10]y_{dec}[n-5] + h[16]y_{dec}[n-8] + \cdots + h[76]y_{dec}[n-38] \quad \textbf{(22)}$$

$$g_{10}[n] = h[1]y_{dec}[n] + h[7]y_{dec}[n-3] + h[13]y_{dec}[n-6] + \cdots + h[79]y_{dec}[n-39] \quad \textbf{(23)}$$

$$g_{11}[n] = h[3]y_{dec}[n-1] + h[9]y_{dec}[n-4] + h[15]y_{dec}[n-7] + \cdots + h[81]y_{dec}[n-40] \quad \textbf{(24)}$$

$$g_{12}[n] = h[5]y_{dec}[n-2] + h[11]y_{dec}[n-5] + h[17]y_{dec}[n-8] + \cdots + h[77]y_{dec}[n-38 \quad (25)$$

Taking inverse Z-transform of the equations (18) to (25), is obtained as

$$G_0[z] = h[0]Y[z] + h[2]z^{-1}Y[z] + h[4]z^{-2}Y[z] + \cdots + h[30]z^{-40}Y[z] \quad (26)$$

$$G_1[z] = h[1]Y[z] + h[3]z^{-1}Y[z] + h[5]z^{-2}Y[z] + \cdots + h[81]z^{-40}Y[z] \quad (27)$$

$$G_{00}[z] = h[0]Y[z] + h[6]z^{-3}Y[z] + h[12]z^{-6}Y[z] + \cdots + h[78]z^{-39}Y[z] \quad (28)$$

$$G_{01}[z] = h[2]z^{-1}Y[z] + h[8]z^{-4}Y[z] + h[14]z^{-7}Y[z] + \cdots + h[80]z^{-40}Y[z] \quad (29)$$

$$G_{01}[z]] = h[4]z^{-2}Y[z] + h[10]z^{-5}Y[z] + h[16]z^{-8}Y[z] + \cdots + h[76]z^{-38}Y[z] \quad (30)$$

$$G_{10}[z] = h[1]Y[z] + h[7]z^{-3}Y[z] + h[13]z^{-6}Y[z] + \cdots + h[79]z^{-39}Y[z] \quad (31)$$

$$G_{11}[z] = h[3]z^{-1}Y[z] + h[9]z^{-4}Y[z] + h[15]z^{-7}Y[z] + \cdots + h[81]z^{-40}Y[z] \quad (32)$$

$$G_{12}[z] = h[5]z^{-2}Y[z] + h[11]z^{-5}Y[z] + h[17]z^{-8}Y[z] + \cdots + h[77]z^{-38}Y[z] \quad (33)$$

For multistage purpose, again the interpolated signal $\{y_{int}[m]\}$ is decimate-by-M=2 and 2 subsequences are interpolated by 3 in the poly-phase branches as a synthesis filter and added together to give the signal $y_{out}[m]$.

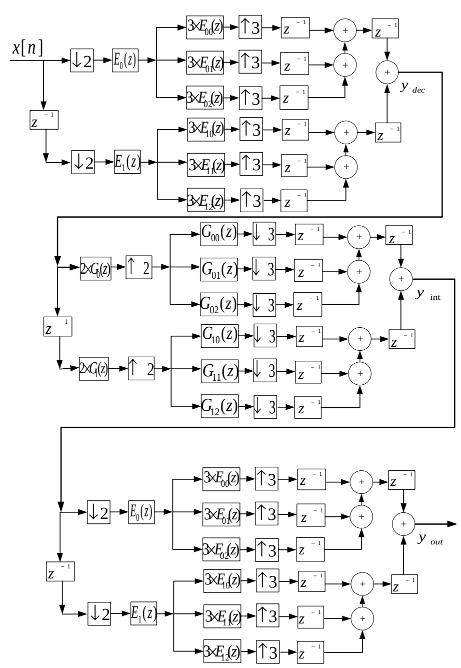



**Figure 6: Proposed Multistage Multirate System for Case 3**

## RESULTS AND DISCUSSIONS

To measure the performance of different noise removal methods, the distortion between original signal and reconstructed signal is measured by the mean square error (MSE), signal-to-noise ratio (SNR) and correlation coefficients values. But in this paper, the performance parameters such as SNR (in dB), MSE, correlation which will prove the robustness of these algorithms are not able to show in values because of lacking of pair to pair signal as shown in

Figure 7. Due to the process, proposed system have used different sampling rate for data conversion and filtering purposes.

Figures 8 to 10 shows the result of a proposed multistage Multirate system using ECG signals in case 1-3 as considered in this paper. All case use multirate filter specification as shown in Table 2-4 on multistage multirate system for remove noisy ECG data. We have considered 1st stage as analysis and $2^{nd}$ stage synthesis filtering part is performed in Multirate system. For multistage operation, we considered repetition process. For case 1, proposed multistage multirate system is considered multirate poly-phase decomposition with decimation and interpolation. The sampling rate conversion factor is considered by down sampling factor M=5 and filter order N=9 for case I. Left side of column in Figure 8-10 (a) shows, output, smooth ECG signal with three different stages of proposed multistage Multirate system for case1-3 and compared with wavelet 9/7 [1],[8] and their perspective frequency histogram. Figure 8 (a), shows better smooth signal where sampling rate is down by M=5 in first stage and the $2^{nd}$ stage increases the sampling rata by L=5 into the multirate polyphase filter. For better performance again, apply $3^{rd}$ stage as analyses filter and got smooth signal than $2^{nd}$ stage as shown in Figure 8 (a). For comparison of the proposed system and wavelet 9/7, we have used histogram analysis and autocorrelation function. Right side column of Figure 8 (a) shows the frequency distribution of noisy ECG signal, smoothed signal by proposed system and wavelet 9/7. The proposed systems have shown better frequency distribution shape where positions are slight because of sampling rate increase or decreases in the system as compared with wavelet 9/7.

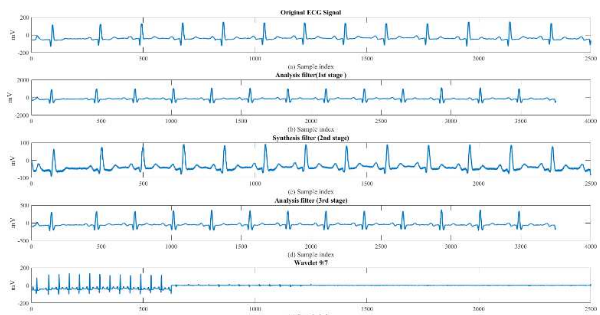
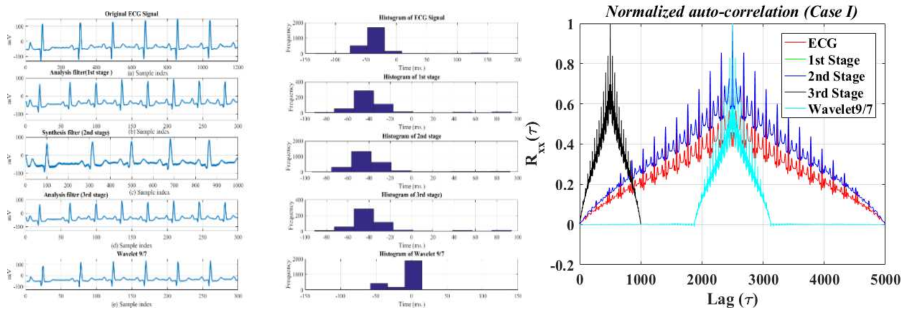



**Figure 7: Multistage Filtering of Noisy ECG Signal (MIT BIH AF 08215m.mat) for Case 1**

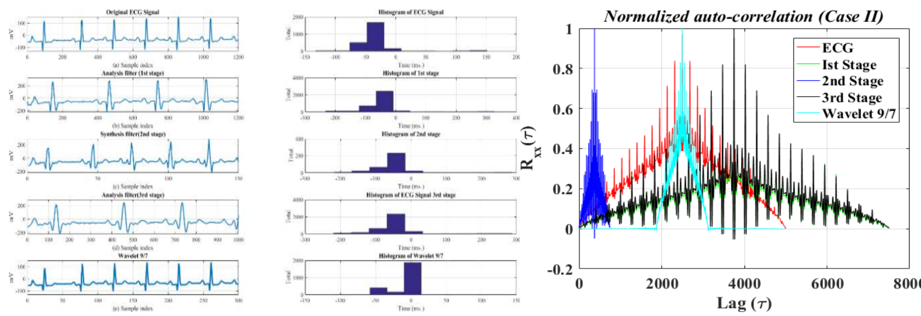
On the other sides in Figure 8 (b) shows a comparison based on autocorrelations. For Case 2, the sampling rate conversion factor is considered by down sampling factor M=2 and filter order N=81. Figure 9 (a), sampling rate is taken for rational sampling factor L/M=3/2 of ECG signals in the analysis filtering part, and show better signal than noisy signal. On the other side, the synthesis filter decreases the sampling rate by M= 10, and get a better signal than analyzed signal ($1^{st}$ stage) as shown in the Figures 9 (a). For better performance again apply analyses filtering by L=10 on that synthesis filtered signal and got a smooth signal than $2^{nd}$ stage as shown in Figure 9 (a). For Case 3, in the analysis, filtering ($1^{st}$ stage) part, sampling rate increased because of rational sampling factor L/M=3/2. The performance between the analysis filter and synthesis filter is also increases the sampling rate L/M=2/3 and gave better results than $1^{st}$ stage as shown in the Figure 10 (a). For better performance apply the output of the $2^{nd}$ stage into the synthesis filtered ($3^{rd}$stage) where the sampling rate was L/M=3/2. The results showed smooth signal than $2^{nd}$ stage.

From the histogram of figure 8-10 (a) for case 1-3, it is also shown that stages 1-3 has a different frequency density with respect to their frequency distribution of various sampling rates and compared with wavelet 9/7. The histogram of 1st stage to $3^{rd}$ stage rates is bell-shaped with one peak for all cases. There are no gaps or extreme values. So, proposed systems give better results than wavelet 9/7 systems for every case.




(**a**) (**b**)

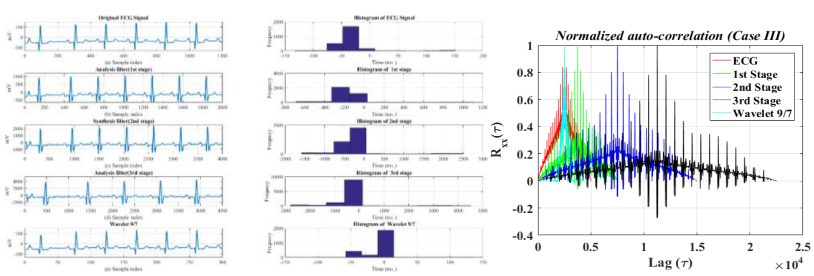
**Figure 8:** (**a**) **Multistage Filtering of ECG Signal** & **Frequency Response** (**b**) Normalized **Autocorrelation for Case 1**




(**a**) (**b**)

**Figure 9:** (**a**) **Multistage Filtering of ECG Signal** & **Frequency Response** (**b**) Normalized **Autocorrelation for Case 2**

Now briefly justify histogram for only case 1, 1$^{st}$ stage has a maximum value of 90 and a minimum value of -85 with one peak between -35 and -60. 2$^{nd}$ stage has a maximum value of 90 and a minimum value of -85, one peak between -35 and -60. 3$^{rd}$ stage has a maximum value of 90 and a minimum value of -85 with one peak between -35 and -60. The wavelet 9/7 has skew left in the frequency distribution where a maximum value of 140 and a minimum value of -110 with one peak between 18 and -12. Such as case 2 & case 3 also show better smooth ECG signal for 1$^{st}$ to 3$^{rd}$ stage than wavelet 9/7.




(**a**) (**b**)

**Figure 10**(**a**): **Multistage Filtering of Noisy ECG Signal (MIT BIH AF 08215m.mat)** & **Frequency Distribution by Histogram** (**b**) Normalized **Autocorrelation for Case 3**

For comparison between the ECG signal and proposed system, we have also considered autocorrelation processes for robustness of algorithms. Figure 8-10(b), the representation of auto correlation values of ECG signal as red color, 1$^{st}$, 2$^{nd}$, 3$^{rd}$ stage as green, blue black color and wavelet 9/7 as cyan color. The results of cases 1-3 with three different stages show high peaks and good correlated values with different sampling rate. Figure 8(b) for case 1 it also observed that the autocorrelation of 1st and 3rd stage overlap to each other because the output of 1st stage is applied to the 2nd stage. Then the output of the 2nd stage is applied to the 3rd stage and finally the L/M is same for 1st & 3rd stage that's why their frequency distribution also same.

## CONCLUSIONS

The novel multistage Multirate system has been proposed for removing noise,requiring low-cost and equal or fractional sampling rates for ECG signal processing. This system has considered three cases for better justification of the proposed system with sampling rate conversion. From the results analysis, it can be concluded that all cases show better quality of ECG signal with the effect of reduce/increase the sample values of noisy ECG signals. These algorithms can be more efficient for diagnosis purpose. In future, we will develop more multistage methodology for biomedical applications to get more accurate performance.

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